

CSC 4504 : Langages formels et applications

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The pq- TRS

Problem 2 --- The pq- TRS

Alphabet = $\{p,q,-\}$

Axiom: for any such x such that x is a possibly empty sequence of ‘-’s,

$xp-qx-$ is an axiom

Generation Rules: for any x,y,z which are possibly empty sequences of ‘-’s,

if $xpyqz$ is a theorem then $xpy-qz-$ is a theorem

A **decision procedure** for a TRS is a process that will always terminate with the correct answer (of whether a given string is a theorem or not)

Question: is there a **decision procedure** for this formal system?

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Why is the pq- TRS practical?

Because it provides us with a formal model of a mathematical property: the addition of integers ---

- $--p---q-----$ is a theorem and “ $2+3=5$ ” is true
- $--p-q--$ is a non-theorem and “ $2+1=2$ ” is false

Problem 2 --- The pq- TRS interpretation

If we interpret

- p as plus
- q as equals
- and a sequence of n ‘-’s as the integer n

then we have

a means of checking $x+y=z$ for all non-negative integers x,y and z

We say that pq- is **consistent** (under the given interpretation) because all theorems are true after interpretation

We say that pq- is **complete** as all true statements (in the domain of interpretation) can be generated as theorems in the system.

We say that the interpretation is **isomorphic** to the system because it is both complete and consistent

Problem 2 --- The pq- TRS extension

The pq- system is isomorphic to a very limited domain of interpretation (but maybe that is all that is required!)

Normally, to widen a domain we can

- add an axiom

- add a generating rule

For example, what happens if we add the axiom:

$$xp - qx.$$

Using this, we can generate many new theorems!

Question: with this new axiom what about completeness and consistency? Can you find an isomorphic interpretation?

Problem 2 --- The tq- system

Question:

- can you define a TRS for modelling the multiplication of two integers
- can you show that it is complete and consistent

Interpretation:

- t as times
- q as equals
- sequences of ‘-’s as integers