Blind Equalization in the Presence of Jammers and Unknown Noise: Solutions Based on Second-Order Cyclostationary Statistics

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***Abstract*—This correspondence addresses the blind identification of a linear time-invariant channel using some second-order cyclostationnary statistics. In contrast to other contributions, the case where the second-order statistics of the noise and of the jammers are totally unknown is considered. It is shown that the channel can be identified consistently by adapting the so-called subspace method of Moulines *et al.* This adaptation is valid for fractionally spaced systems and, more interestingly, for the general systems exhibiting transmitter induced cyclostationnarity introduced by Tsatsanis and Giannakis. The new subspace method is based in both cases on a common tool, i.e., a general spectral factorization algorithm. The identifiability conditions are specified and some simulation examples are given.**

I. Introduction

Under standard hypotheses (linear modulation, time-invariant channel), the complex envelope of the (noise-free) received signal in a digital communication context is[1] $x_a(t) = \sum_{k\in\mathbb{Z}} s(k)h_a(t-kT)$, where

$\{s(n)\}$ sequence of symbols;
T symbol period;
$h_a(\cdot)$ composite time-limited causal mapping;

the support of which is $[0, M_aT]$, say, accounting for shaping, the multipath effects, and the reception filter. It is easily checked that $x_a(t)$ can be rewritten as

$$x_a(t) = \sum_{k\in\mathbb{Z}} \tilde{s}(k)h_a\left(t - k\frac{T}{q}\right) \tag{1.1}$$

where $q \geq 1$ is any integer, and $\{\tilde{s}(n)\}$ is the so-called zero-padding sequence at the rate $\frac{q}{T}$, given by $\tilde{s}(qn) = s(n)$ and $\tilde{s}(nq+k) = 0$ for $k = 1\cdots q-1$. Hence, the expression of the sampled version of $x_a(t)$ at the rate $\frac{q}{T}$ is

$$x(n) = [h(z)]\cdot\tilde{s}(n) \tag{1.2}$$

where $h(z) = \sum_{k=0}^{qM_a} h_k z^{-k}$, and $h_k = h_a(k\frac{T}{q})$ (without any restriction, we have imposed qM_a to be an integer). Thus, the sampled observation is the output of an unknown finite impulse response filter $h(z)$ of degree qM_a driven by the sequence $\{\tilde{s}(n)\}$, leading to inter-symbol-interference (ISI). It is of interest to identify the unknown channel $h(z)$ from the second-order statistics of the observation to remove the ISI. Concerning this point, the model (1.2) calls for the following observations.

- If $q = 1$ (standard systems), $\tilde{s}(n) = s(n)$, and in general, the second-order statistics of $\{x(n)\}$ do not allow the identification of $h(z)$.
- *A contrario*, the case $q > 1$, which corresponds to a fractional sampling (FS) system, deserves consideration. Noticing that $\{\tilde{s}(n)\}$ is cyclostationary at the cyclic frequencies $0, \ldots, \frac{q-1}{q}$, it was proved indeed that if $h(z)$ does not possess q zeros on a circle separated by $\frac{2\pi}{q}$ radian angles, the *entire* second-order statistics of $\{x(n)\}$ enable the identification of $h(z)$. Various time-domain estimation algorithms of $h(z)$ based on the second-order statistics of the observation have been proposed in [7], [15], and [14]. These approaches can be extended to the case where the useful signal $x_a(t)$ is corrupted by an additive noise or/and interferences with known (up to a scalar factor) second-order statistics.

The purpose of this correspondence is to develop a simple blind identification scheme, relying on *certain second-order statistics*, which remain consistent when $x_a(t)$ is corrupted by an additive noise or/and interference process $i_a(t)$, uncorrelated with $x_a(t)$, the second-order statistics of which are *unknown*.

In the case of an FS system with $q > 2$, it was briefly remarked by Giannakis in [8] that it is possible to identify $h(z)$ from the cyclostatistics of the noisy observed signal[2] $x(n)+i(n)$ at the nonnull cycles $1/q, \ldots, (q-1)/q$, provided that these cycles are not cycles of $\{i(n)\}$.

This principle of separating the contributions of the different cycles and then removing the corrupted statistics is clever as far as the struggle against jammers is concerned. One should nevertheless note that this theoretical approach may prove useless in certain FS contexts since the bandlimited character of a communication channel makes most of the cyclo-statistics of interest numerically negligible, and the aforementioned method is often prone to degeneracies (see [1]–[3]). In order to deal with numerically significant cycles, one idea is to *impose* some second-order cyclic properties at the emitter: the so-called concept of transmitter induced cyclostationarity (TIC) was introduced in [11] and has met with various extensions since (see, e.g., [10], [12], and [13]).

The principle of TIC is to transmit a sequence of pseudo-symbols $\{v(n)\}$ at a larger rate $(\frac{1}{T'})$ than that of the original symbol sequence[3]. The transmission is such that the noise-free analog signal so received can be written as

$$x_a(t) = \sum_{k\in\mathbb{Z}} v(k)h_a(t-kT'). \tag{1.3}$$

The sampled version at the rate $\frac{1}{T'}$ is then

$$x(n) = [h(z)]\cdot v(n). \tag{1.4}$$

This time, $h(z) = \sum_{k=0}^{\frac{T}{T'}M_a} h_k z^{-k}$, and $h_k = h_a(kT')$ where, as usual, $\frac{T}{T'}M_a$ is assumed to be an integer. This formulation shows that (1.4) is a direct generalization of (1.2) in which $v(n) = \tilde{s}(n)$ and $T' = T/q$. The reader may wish to consult the various contributions to appreciate the communication-oriented problems inherent to TIC systems.

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[1] The subscript "a" stands for "analog."

[2] $i(n)$ is the sampled version of $i_a(t)$ at the rate $\frac{q}{T}$.

[3] The one-to-one correspondence between $\{v(n)\}$ and $\{s(n)\}$ is obviously assumed.

Dealing with the model (1.4), we propose here to show that the subspace method of [5] can be adapted to identify $h(z)$ from some reliable statistics, namely, those free of any corruption. In Section II, a general spectral factorization algorithm is presented. Section III applies this algorithm to blind identification; rather than developing a general method, we focus on three particular cases: the FS case, the repetition coding case [11], and the modulation case [13]. Extensions to other contexts are possible. In each scenario, we thoroughly depict the spectral factorization and make some remarks on the identifiability conditions. Simulation examples are subsequently given and analyzed. Section IV summarizes the main points previously studied for various TIC systems.

II. A Factorization Algorithm

Let $S(z)$ be a $q \times 1$ rational function[4] of the form

$$S(z) = H(z) l^*(z^{-1}) \tag{2.5}$$

where $H(z) = \sum_{k=0}^{M} H_k z^{-k}$ is a $q \times 1$ degree M polynomial of the variable z^{-1}, $l(z)$ is a scalar-valued[5] causal rational transfer function, and $l^*(z)$ is obtained by conjugating the coefficients of $l(z)$. Of course, $S(z)$ has a Laurent expansion $S(z) = \sum_{k \in \mathbb{Z}} S_k z^{-k}$ converging around the unit circle. We focus on the problem of retrieving $H(z)$ from $S(z)$. As $H(z)$ is a polynomial, it is clear that $S_k = 0$ as soon as $k > M$, and we consequently consider

Problem 2.1: Given $N \geq M$, under which conditions is it possible to recover $H(z)$ from the Laurent coefficients $\{S_k\}_{k=-N,\ldots,N}$ of $S(z)$? Exhibit a means of extracting $H(z)$.

We assume the following hypotheses:

- $\mathbf{H1}$ $H(z) \neq 0$ for each z, including ∞.
- $\mathbf{H2}$ $l_0 = l(\infty)$ is nonzero.

Problem (2.1) could be solved by developing a linear prediction-like method. However, for simplicity, we shall rather generalize the noise subspace approach of [7].

We first need to recall an important result presented in [5]. Let $\mathcal{B}_N$ be the set of all $q(N+1)$-dimensional row vectors $G = [G_0, \ldots, G_N]$ satisfying the linear relation $G(z)H(z) = 0$, where $G(z) = \sum_{k=0}^{N} G_k z^{-k}$. $\mathcal{B}_N$ is, of course, a linear subspace of $\mathbb{C}^{q(N+1)}$. Let Π_N be the orthogonal projector onto $\mathcal{B}_N$. For every $q \times 1$ polynomial $A(z) = \sum_{k=0}^{M} A_k z^{-k}$ of degree M, we denote by $\mathcal{T}_N(A)$ the $q(N+1) \times (N+M+1)$ Sylvester matrix defined as

$$\mathcal{T}_N(A) = \begin{bmatrix} A_0 & A_1 & \ldots & A_M & 0 & \ldots & 0 \\ 0 & A_0 & A_1 & \ldots & A_M & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \ldots & 0 & A_0 & A_1 & \ldots & A_M \end{bmatrix}. \tag{2.6}$$

Then, we have the following result:

Theorem 2.1: Let $F(z)$ be a $q \times 1$ polynomial of degree M. If $\mathbf{H1}$ is true, then, for $N \geq M$, the linear equation

$$\Pi_N \mathcal{T}_N(F) = 0 \tag{2.7}$$

holds if and only if $F(z)$ coincides with $H(z)$ up to a scalar factor.

In other words, $H(z)$ can be identified from the subspace $\mathcal{B}_N$ by solving a linear system. This follows from properties of minimal polynomial bases of a rational subspace; see [4] for details. It now remains to be shown that $\mathcal{B}_N$, and hence Π_N, can, under $\mathbf{H2}$, be extracted from the Laurent coefficients $\{S_k\}_{-N,\ldots,N}$ of $S(z)$. It is clear that $G(z) = \sum_{k=0}^{N} G_k z^{-k}$, of dimension $1 \times q$, satisfies $G(z)H(z) = 0$ if and only if $G(z)S(z) = 0$. Denoting by $\mathcal{S}_N$ the $q(N+1) \times (2N+1)$ matrix defined as

$$\mathcal{S}_N = \begin{bmatrix} S_0 & \ldots & S_N & & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \\ S_{-N} & \ldots & S_0 & \ldots & S_N \end{bmatrix}$$

$G(z)S(z) = 0$ implies in particular that $G\mathcal{S}_N = 0$, which is equivalent to the condition $[G(z)S(z)]_- = 0$, where the notation $[\cdot]_-$ stands for the causal truncation of the function inside the brackets. Conversely, $[G(z)S(z)]_- = 0$ also means that $[k(z)l^*(z^{-1})]_- = 0$, where $k(z)$ is the scalar-valued polynomial $G(z)H(z)$. Assuming now that $\mathbf{H2}$ holds, we deduce immediately that $k(z) = 0$. Hence, the space $\mathcal{B}_N$ coincides with the left kernel of matrix $\mathcal{S}_N$ associated with $S(z)$.

Problem (2.1) is solved. We have found a method for recovering $H(z)$, which is valid as long as $\mathbf{H1}$ and $\mathbf{H2}$ hold. We now recast some blind second-order problems into this framework.

TABLE I
FS: Realization of the Paths

	delays	attenuation
1^{st} path	$0.0000T$	1
2^{nd} path	$0.2084T$	1
3^{rd} path	$0.8880T$	1

TABLE II
FS: Polyphase Components of the Channel

$h_1(n)$	$h_2(n)$	$h_3(n)$
0.0415 + 0.0095i	0.1393 - 0.2058i	0.5407 - 0.1906i
0.1499 - 0.0171i	0.2099 - 0.2797i	0.3819 - 0.0896i
0.1794 - 0.0971i	0.4252 - 0.2715i	0.1096 - 0.0188i

III. Application to Blind Identification

Generally speaking, the correlation coefficient at lag τ and at cycle α of a second-order process $\{y(n)\}$ is defined as (see [16] and the references therein)

$$R_y^{(\alpha)}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} E[y(n+\tau)y(n)^*] e^{-i2\pi\alpha n} \tag{3.8}$$

and the corresponding cyclo-spectrum is defined as $S_y^{(\alpha)}(e^{i\omega}) = \sum_\tau R_y^{(\alpha)}(\tau) e^{-i\omega\tau}$.

According to model (1.4), it is easy to prove that the expression of the cyclo-spectrum of $\{x(n)\}$ at cycle α is

$$S_x^{(\alpha)}(e^{i\omega}) = h(e^{i\omega}) h(e^{i(\omega - 2\pi\alpha)})^* S_v^{(\alpha)}(e^{i\omega}). \tag{3.9}$$

The expression (3.9) remains valid in a noisy context, as soon as the noise and the jammers are decorrelated from the signal of interest and do not admit α as a cycle.

Although it is possible to develop further the general framework depicted thus far, such would result in cumbersome relations. We shall thus concentrate henceforth on the three cases raised in the introduction.

A. The FS Case

It is assumed here that $q > 2$ so that at least two nonnull cycles can be exploited. The contribution of the jammers $\{i(n)\}$ is assumed not to exhibit cyclostationarity at the shifts $\frac{1}{q}, \ldots, \frac{q-1}{q}$, and we recall that $\{v(n) = \tilde{s}(n)\}$, and $T' = T/q$. We propose to develop a scheme in the case where the input symbols are white; the method

[4] In the sequel, capital letters stand for matrices or vectors.

[5] the case when $l(z)$ is vector-valued can also be treated this way.

TABLE III
MODULATION: TEN CHANNEL REALIZATIONS

$h(0)$	$h(1)$	$h(2)$	$h(3)$	$h(4)$
0.0766 - 0.0472i	0.1727 + 0.2774i	-0.5916 + 0.2153i	-0.2405 + 0.6518i	-0.0645 - 0.0394i
-0.0300 + 0.0204i	0.1597 - 0.0300i	0.4166 + 0.7988i	-0.1903 + 0.3093i	-0.1673 + 0.0215i
0.4175 - 0.1257i	0.1802 + 0.3764i	-0.6259 + 0.3107i	-0.3777 + 0.0284i	0.0128 + 0.0591i
0.8130 + 0.0577i	-0.2574 - 0.1393i	-0.0456 - 0.2748i	0.0356 + 0.3854i	-0.0951 - 0.1151i
-0.0402 + 0.0787i	0.2298 - 0.1537i	0.2845 - 0.0574i	-0.4862 + 0.7622i	0.0265 - 0.1149i
0.2502 + 0.0668i	0.8357 + 0.4010i	-0.1944 + 0.0851i	-0.1363 + 0.0901i	0.0324 - 0.0279i
0.3149 - 0.0940i	-0.1115 - 0.1508i	0.1901 - 0.8604i	-0.0521 - 0.2384i	-0.0681 + 0.1261i
-0.0383 - 0.0137i	0.4540 + 0.0190i	0.1122 - 0.6541i	0.3695 - 0.4206i	-0.1635 + 0.1052i
0.1348 - 0.1371i	0.6038 - 0.0735i	-0.5815 + 0.0650i	-0.4808 - 0.0372i	0.1239 + 0.0512i
-0.0251 - 0.0294i	0.5445 + 0.46879	0.1939 + 0.4547i	-0.4566 + 0.1545i	0.0738 + 0.0024i

can be extended directly when the symbols are colored. Under this assumption, $S_v^{(\alpha)}(e^{i\omega}) = \frac{1}{q}$ for all $\alpha = \frac{1}{q}, \ldots, \frac{q-1}{q}$. Consider now the following vector:

$$S(e^{i\omega}) = q\left[S_x^{(1/q)}(e^{i\omega})^*, \ldots, S_x^{((q-1)/q)}(e^{i\omega})^*\right]^T.$$

Recalling (3.9) and defining

$$H(z) = \left[h\left(ze^{\frac{i2\pi}{q}}\right), \ldots, h\left(ze^{\frac{i2\pi(q-1)}{q}}\right)\right]^T$$

and $l(z) = h(z)$, the factorization (2.5) holds. The results of the previous section then apply. Provided $h(z)$ does not possess $q-1$ zeros on a circle, equally separated by $\frac{2\pi}{q}$ radians, one can identify $h(z)$ from the cyclo-statistics of the observation at the nonnull cycles. In practice, the Laurent coefficients of $S(z)$ are unknown, but they can be replaced by consistent estimates in the procedure sketched in Section II [(2.7) should be solved in the least squares sense]. The proposed estimate of the channel is, of course, consistent.

Let us now consider the practical aspects of the above-mentioned approach. In order to enlighten the identifiability condition, one may consider increasing the oversampling rate; indeed, the more cycles, the less stringent the identifiability condition. However, the order M increases with q, and since the analog filter $h_a(t)$ is bandlimited, a larger q reduces the bandwidth of $h(z)$; in other words, according to (3.9), there are only two nonnull numerically relevant cycles: $\frac{1}{q}$, $\frac{q-1}{q}$.

A good practical choice is $q = 3$. The question is: What is the relevance of removing the zero cycle? When the problem is well-conditioned (this occurs for a large excess bandwidth and for short impulse responses), the color of the noise does not significantly impact the performance of the full method (all cycles considered; see [7]), even if consistency is lost. By contrast, consistency is crucial in ill-conditionned problems. In this case, the full method is very sensitive to the perturbations brought by the color of the noise and fails to provide a good estimate. Excluding the zero cycle is then recommended; of course, the bad conditioning compels one to use a large analysis window.

Simulation Results: $q = 3$. We consider a GSM channel: the shaping is a raised-cosine with 85% excess bandwidth; the symbol period is $T = 3.7\,\mu$s, and the carrier frequency is $f_0 = 1.087\,654$ GHz. A three-path realization is studied. The characteristics of the channel are given in Tables I and II. The digital channel $h(z)$ is consequently a degree $M = 8$ polynomial. The symbols are BPSK. The colored noise is the output of $r(z) = \frac{1}{\sqrt{3}}(1 + z^{-2} + z^{-4})$ driven by a white Gaussian noise independent of the symbols. The averaged square Euclidean distance between the estimate and the true channel (MSE) is estimated from 200 Monte Carlo trials. In Fig. 1, $N = 1000$, and the SNR goes from 4 to 16 dB. As expected, removing the zero cycle is all the less pertinent as the SNR increases.

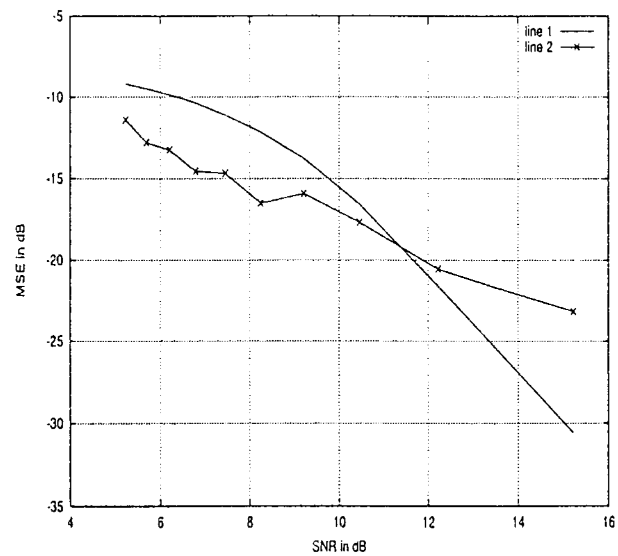

Fig. 1. FS system. Line 1: standard method. Line 2: 0 cycle excluded; $N = 1000$.

B. The Repetition Coding and Interleaving Scheme [11]

One now transmits the sequence $\{v(n)\}$ at the rate $\frac{1}{T'} = \frac{2}{T}$, which may be read as consecutive blocks of the type $[s(nL), s(nL+1), \ldots, s(nL+L-1) \| s(nL), s(nL+1), \ldots, s(nL+L-1)]$. It is easy to prove that $\{v(n)\}$ admits $(\frac{2k+1}{2L})_{k=0,\ldots,L-1}$ as nonzero cyclic frequencies.

Let us, moreover, assume that $\{s(n)\}$ is white. Setting $\mu_k = 1/(L(1 - e^{-i2\pi(2k+1)/2L}))$, a simple computation gives

$$S_v^{(2k+1/2L)}(z) = \mu_k(z + z^{-1}),\ \mu_k(z^L + z^{-L}).$$

Consider now the following vector

$$\tilde{S}(e^{i\omega}) = \left[\frac{1}{\mu_{k_1}^*} S_x^{((2k_1+1)/2L)}(e^{i\omega})^*, \ldots, \frac{1}{\mu_{k_N}^*} S_x^{(2k_N-1/2L)}(e^{i\omega})^*\right]^T$$

for any collection of N distinct $k_i \in \{1, \ldots, L\}$. This yields $\tilde{S}(z) = z^{-L}H(z)l^*(z^{-1})$, where

$$H(z) = \left[h(ze^{i2\pi(2k_1+1)/2L}), \ldots, h(ze^{i2\pi(2k_N+1)/2L})\right]^T$$

and $l(z) = (1 + z^{-2L})h(z)$. The results of Section II still hold if one considers $S(z) = z\tilde{S}(z)$. Noticing that L is a design parameter and can then be chosen arbitrarily big, two crucial remarks follow:

- *Many Cycles in the Factorization:* Let us exploit all the nonnull cycles so that there are $L-1$ entries in $H(z)$. On the one

TABLE IV
TIC AND BLIND SECOND-ORDER IDENTIFICATION

	FS at $3/T$	repetition	modulation
support of $h_a(t)$	M_aT	M_aT	M_aT
degree of $h(z)$ to identify	$M = 3M_a$	$M = 2M_a$	$M = M_a$
number of cycles used	$p = 2$	$2 \leq p < L$	$p \geq 2$
identifiability condition	$h(z_0) = 0 \Rightarrow h(z_0 e^{i2\pi/3}) \neq 0$	none	none
control of cycles	no	yes	yes
estimation in jammers	consistent	consistent	consistent
generalization to colored sources	yes	yes	yes
over-estimation of the degree	no	yes	yes

hand, the identifiability condition is that $h(z)$ does not possess $L-1$ zeros on a circle equally separated by $\frac{2\pi}{L}$. On the other hand, $h(z)$ has M zeros. If one imposes $L > M$, the previous points are contradictory, hence showing that the identifiability condition is automatically fulfilled (see [11]).

- *Two Well-Chosen Cycles:* Take any two cycles $\frac{2k_1+1}{2L}$ and $\frac{2k_2+1}{2L}$ so that $k_2 - k_1$ and L are coprime (this is always possible since L can be chosen arbitrarily large). According to a structural result of [13] (see also [10]), the identification of $h(z)$ is possible, without any restriction on $h(z)$, as soon as $L > M$.

Therefore, the identification in a repetition context is robust as regards the unknown channel.

C. The Modulation Case

In this model, $T' = T$, and the sequence $\{v(n) = f(n)s(n)\}$ is transmitted, where $f(n)$ is a deterministic (almost) periodic sequence. This scheme has been proposed independently in [10] and [13]. For sake of simplicity, we restrict the study to i.i.d. sequences of symbols[6]. Under this condition, it is easy to prove that $S_v^{(\alpha)}(z) = \lambda_\alpha$ for some α and λ_α, depending on the development of $f(n)$ as a Fourier series. Suppose α_1 and α_2 are nonnull cycles. Consider the function[7]

$$S(e^{i\omega}) = \left[\frac{1}{\lambda^*_{\alpha_1}} S_x^{(\alpha_1)}(e^{i\omega})^*, \frac{1}{\lambda^*_{\alpha_2}} S_x^{(\alpha_2)}(e^{i\omega})^*\right]^T.$$

If $H(z) = [h(ze^{i2\pi\alpha_1}), h(ze^{i2\pi\alpha_2})]^T$ and $l(z) = h(z)$, we have $S(z) = H(z)l^*(z^{-1})$.

As in the previous section, the identifiability condition vanishes in the following two cases (see [10] and [13]):

- $\alpha_2 - \alpha_1$ is irrational, or
- $\alpha_2 - \alpha_1 = \frac{k}{p}$, k and p coprime, is such that $p > M$, M being as usual the degree of $h(z)$.

As compared with the repetition scheme, the modulation of the symbols also brings a robust way of identifying the channel. The advantage of modulation over repetition is that the channel order is halved, thus yielding faster algorithms, with a lower computational burden.

Simulation Results: The pulse is a raised-cosine with 20% excess bandwidth; $T = 3.7$ μs, and $f_0 = 1.087\,654$ GHz. We averaged over ten realizations of channels resulting from five multipaths. The maximum delay is set to $2T$ so that the degree is $M = 4$. The impulse responses are in Table III. The deterministic sequence is $f(n) = \frac{1}{\sqrt{1+\gamma^2}}(1 + \gamma e^{i2\pi\alpha n})$, with $\gamma = 0.5$ and $\alpha = \frac{51}{360}$. The symbols are white BPSK sequences. In the factorization algorithm, we chose the two cyclo-frequencies α and $-\alpha$. In Fig. 2, the MSE is given as the number of observed samples increases. The SNR is set to 10 dB; the noise is either white Gaussian or is the output of $r(z) = \frac{1}{\sqrt{3}}(1 + z^{-2} + z^{-4})$ driven by a white Gaussian noise. Notice

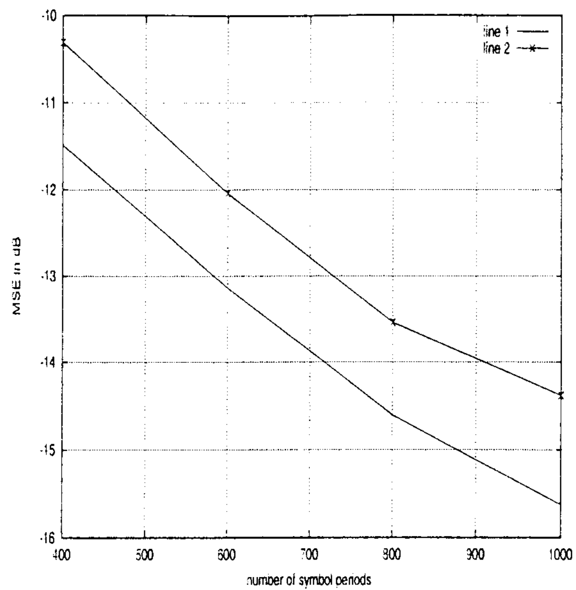

Fig. 2. Modulation. Line 1: white noise. Line 2: colored noise; $\mathrm{SNR} = 10$ dB.

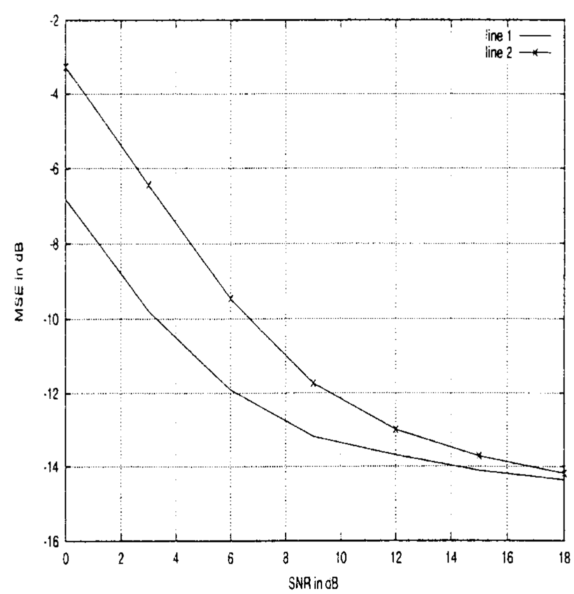

Fig. 3. Modulation. Line 1: white noise. Line 2: colored noise; $N = 600$ samples.

[6] For a generalization to any distribution, see [13].

[7] More cycles may be taken into account.

the consistency of the estimate when the noise has an unknown color. In Fig. 3, the observation lasts 600 symbol periods. The SNR varies from 0 to 18 dB.

The results thus obtained are satisfactory; however, the “best” choice of a sequence $f(n)$ is currently under investigation.

IV. Conclusion

When cyclostationarity is induced at the transmitter, we have shown that some second-order cyclo-spectra provide a way of identifying the unknown channel. In contrast to the conventional approches, the consistency of the proposed method is achieved when the observation is corrupted by interference, the second-order structures of which are unknown. We propose in Table IV a summary of the main points developed in the paper.

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Memory Efficient Programmable Processor Chip for Inverse Haar Transform

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***Abstract*— In this correspondence, a processor chip programmable between $N = 8$ and $N = 1024$ for the unidimensional inverse Haar transform (1-D-IFHT) is presented. The processor uses a low latency data-flow with an architecture that minimizes the internal memory and an adder/subtracter as the only computing element. The control logic has a single and modular structure and can be easily extended to longer transforms. A prototype of the 1-D-IFHT processor has been implemented using a standard-cell design methodology and a 1.0-μm CMOS process on a 11.7 mm^2 die. The maximum data rate is close to 60 MHz.**

***Index Terms*—Digital signal processors, Haar transforms, image processing, transform coding, very-large-scale integration.**

I. Introduction

Modern digital communications systems require efficient data coding in order to reduce transmission and/or storage costs [1], [2]. These coding systems use algorithms of one of the following types: waveform coders, transform coders, and model coders [2]. Although transform coding systems generally use the cosine transform [3], [4], the use of the Haar transform offers certain advantages over the former due to the wavelet characteristics of Haar functions [5].

Three generic VLSI architectures have been proposed in [6] for the hardware implementation of the Haar transform. Each of these has different characteristics in terms of computation time, complexity, input/output format, and pipelinability. Some other important characteristics not considered in [6] are the size of the required memory, the input ordering, and the complexity of control. The selection of one architecture or another depends on the application and the design framework available so that in some cases, the resulting architecture has a structure that is a hybrid of other more general structures. This is the case of the direct Haar transform processor described in [7], whose architecture consists of three stages in pipeline, the last of these having sequential queue architecture.

The on-line computing of the bidimensional Haar transform can be carried out directly by implementing the two-dimensional (2-D) fast transform [8], [9] or indirectly by applying the property of separability [2] using the unidimensional (1-D) transform. The three processor parallel-pipeline architecture described in [8] has been designed for a raster ordering of input, minimizing the internal memory. The simplicity of the architecture proposed by Albanesi and Ferreti [9] is due to the fact that the input data ordering is not a raster ordering and that they use a less general bidimensional Haar-like transform.

This correspondence presents an inverse Haar transform processor (1-D-IFHT) programmable for transform lengths of between $N = 8$ and $N = 1024$. Its architecture, which is a variant of the sequential queue architecture proposed in [6], reduces the internal memory requirement from N to $\log_2 N$. Moreover, the control logic has a modular structure whose complexity increases linearly with $\log_2 N$.

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