# Comments and Replies

## Comments on "Conjugate ESPRIT (C-SPRIT)"

Jean-Pierre Delmas

*Abstract*—**It is shown that in the paper by N. Tayem and H. M. Kwon, the Esprit-like algorithm was derived from a rather unrealistic assumption, and therefore, this conjugate Esprit algorithm will have little utility in practice.**

*Index Terms*—**Antenna array, direction of arrival (DOA) estimation, noncircular signals.**

In [1], the authors present an Esprit-like algorithm to estimate the directions of arrivals (DOA) from noncoherent 1-D signal sources such as binary phase shift keying (BPSK) and M-ary amplitude shift keying (MASK). The proposed algorithm provides more precise DOA estimates and can detect more signals than well-known classical subspace-methods MUSIC and ESPRIT for 1-D signals. The paper has been read with great interest; however, it appears to be based on a key assumption that is never satisfied in practice. In this note, we explain the correct model that must be used for estimation of DOA of narrowband (1-D) signal sources.

The authors consider a uniform linear array composed of $M$ elements that receives $K$ noncoherent and narrowband 1-D signals from different DOAs $\theta_k, k = 1, \ldots, K$. The $M \times 1$ received signal satisfies the following standard model

$$\mathbf{y}_t = \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_{k,t} + \mathbf{n}_t$$

where the first component of all the steering vectors $\mathbf{a}(\theta_k)$ is equal to one, and where it is assumed in [1] that $s_{k,t}$, $k = 1, \ldots, K$ are real-valued for 1-D signals such as BPSK and MASK. The two assumptions

$$[\mathbf{a}(\theta_k)]_1 = 1 \quad \text{and} \quad s_{k,t} = s^*_{k,t}, \quad \text{for } k = 1, \ldots, K \tag{1}$$

are key assumptions that allow the authors to prove the following [1, Eq. (15)]:

$$\begin{pmatrix} \mathbf{y}_{t,1} \\ \mathbf{y}_{t,2} \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{A}\mathbf{\Phi}^* \end{pmatrix} \mathbf{s}_t + \begin{pmatrix} \mathbf{n}_{t,1} \\ \mathbf{n}_{t,2} \end{pmatrix}$$

on which the proposed conjugate ESPRIT is devised.

We prove in the following that the two constraints (1) are not consistent for one-dimensional signals. In practice, the DOA estimation is made after frequency down conversion: The analog received BPSK or MASK modulated signals are bandpass filtered and after down-shifting the signal to baseband, the in-phase and quadrature components are matched-filtered, sampled and paired to obtain complex signals $s_{k,t}$. The output of the matched filter associated with the first element of the array is represented as a sum $\sum_{k=1}^{K} s_{k,t}$ of 1-D signals with

$$s_{k,t} = \alpha_{k,t} e^{i\phi_k}$$

where $\alpha_{k,t}$ are real-valued and where $\phi_k$ are *arbitrary phase shifts* that can be different for each signal but constant with time. Consequently, the *amplitudes* $s_{k,t}$ *are complex-valued*. This model has been used by numerous authors who have studied DOA estimation of non-circular sources (see, e.g., [2, Eqs. (5), (7)], [3, Eq. (28)], [4, Eq. (2)], [5, Subsect. II.A], [6, p. 2, bottom)]). Note that in these references all emphasize that the phases $\phi_k$ are arbitrary phase shifts (denoted natural phases in [2], [3]) that can be different for each signal.

The frequency offsets can be naturally neglected, but the phases are *intrinsic to the model if the first component of the steering vector is normalized to one*. The assumption that $s_{k,t}$ are all real-valued would correspond to the case where all the $K$ source signals with the same carrier frequency would be located at distances $d_k$ from the reference element, which in turn would be a multiple of the wavelength associated with the carrier frequency: It is clear that this assumption is completely unrealistic.

This assumption cannot be circumvented because if phases were introduced into relations (12) of [1], the proposed algorithm would no longer be valid. Finally we note that this assumption (1) has been also used in very numerous papers of the authors dedicated to non-circular signals (see e.g., [7]–[9] for coherent non-circular sources for which the different signals are phase delayed amplitude weighted replicas on one of them), [10] for 2-D root MUSIC algorithms and [11] for Toepliz based matrix pencil. Consequently all the nice results given in these papers are misleading because they do not apply in practice.

Manuscript received July 11, 2006.

The author is with the GET/INT, Département CITI, UMR-CNRS 5157, Institut National des Télécommunications, 91011 Evry Cedex, France (e-mail: jean-pierre.delmas@int-evry.fr).

Digital Object Identifier 10.1109/TAP.2006.889995

### References

[1] N. Tayem and H. M. Kwon, "Conjugate ESPRIT (C.SPRIT)," *IEEE Trans. Antennas Propag.*, vol. 52, no. 10, pp. 2618–2624, Oct. 2004.
[2] P. Chargé, Y. Wang, and J. Saillard, "A non-circular sources direction finding method using polynomial rooting," *Signal Processing*, vol. 81, pp. 1765–1770, 2001.
[3] A. Zoubir, P. Chargé, and Y. Wang, "Non circular sources localization with Esprit," presented at the Eur. Conf. Wireless Technology, Munich, Germany, Oct. 6–10, 2003.
[4] M. Haardt and F. Römer, "Enhancements of unitary Esprit for non circular," presented at the Proc. IEEE Int. Conf. ASSP, Montreal, Canada, May 17–21, 2004.
[5] P. Chevalier and F. Pipon, "New insights into optimal widely linear array receivers for the demodulation of BPSK, MSK and GMSK signals corrupted by non circular interferences—Application to SAIC," *IEEE Trans. Signal Processing*, vol. 54, no. 3, pp. 870–883, Mar. 2006.
[6] H. Abeida and J. P. Delmas, "MUSIC-like estimation of direction of arrival for non-circular sources," *IEEE Trans. Signal Processing*, vol. 54, no. 7, pp. 2678–2690, Jul. 2006.
[7] A. Salamech and N. Tayem, "Conjugate MUSIC for non-circular sources," *IEEE Trans. Antennas Propag.*, submitted for publication.
[8] N. Tayem and A. Salamech, "Angle of arrival estimation for non-circular signals," presented at the IEEE Sarnoff Symp., Princeton, NJ, Mar. 21–25, 2006.
[9] A. Salamech and N. Tayem, "Conjugate MUSIC for non-circular sources," presented at the Proc. IEEE Int. Conf. ASSP, Toulouse, May 15–16, 2006.
[10] A. Salamech, N. Tayem, and H. K. Kwon, "Improved 2-D root MUSIC for non-circular signals," presented at the Workshop on Sensor Array and Multichannel (SAM), Waltham, MA, Jul. 12–14, 2006.
[11] N. Tayem, A. Salamech, and H. K. Kwon, "Toepliz based matrix pencil for non-circular signals," in *Proc. IEEE Vehicular Technology Conf.*, Montreal, Canada, Sep. 25–28, 2006.