Capacity Requirements in Networks of Quantum Repeaters and Terminals

Michel Barbeau[1] Joaquin Garcia-Alfaro[2]
Evangelos Kranakis[1]

[1]Carleton University

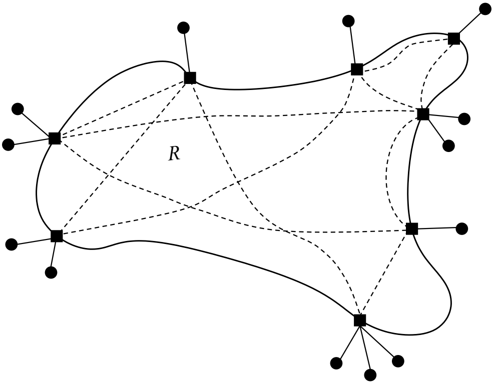
[2]Institute Polytechnique de Paris

October 13, 2020

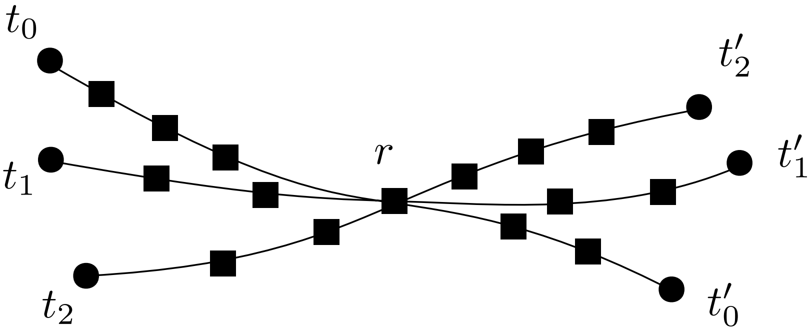
Outline of the work

- Topic: Path congestion avoidance in networks of quantum repeaters and terminals
- Assumption: Complete paths between terminals
- What is the required quantum memory size in repeaters?
- Contributions:
    - Lower and upper bounds for the required qubit memory size of repeaters for general graphs and two-dimensional grid network topologies
    - Congestion avoidance algorithm: Layer-peeling path establishment

Network Model



Repeater r capacity



$C_P(r)$ is the number of supported paths

Repeater Model

- Simple error model: single qubit errors in Bell-EPR pairs
- Achieve fidelity with purification
- Adjacent nodes use direct communications to establish entanglement
- Remote nodes use entanglement swapping and teleportation
- Quantum memory size of a repeater is equal to the sum of the lengths of the paths going through it (Lemma 7)

Evaluation Metrics

- For each simulation, we compute the following metrics
  - **Congestion:** # of paths passing through most visited repeater
  - **Entanglement rate:** Following existing work (cf. [24,25,26])

$$\mathcal{T}(n) = \begin{cases} 1/R(n), \text{ if } \mathcal{X}_{ch} \geq \tau(n) - (\mathcal{X}_s - \tau(1)) \\ 0, \text{ else} \end{cases}$$

(precise calculation is summarized in the paper)

General Graphs

- Minimum required quantum memory (Corollary 9)

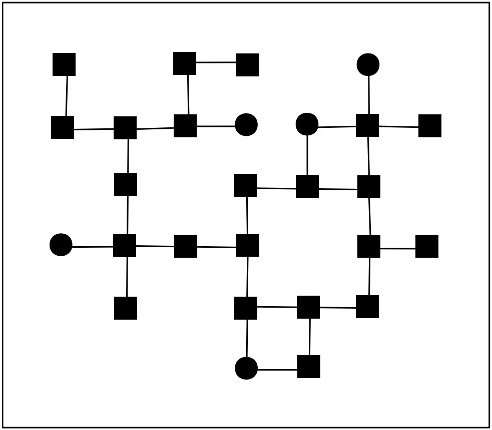
$$M_P(r) \geq 2 \left\lceil \frac{1}{|R|} \binom{|T|}{2} \right\rceil \text{ qubits}$$

- Maximum required quantum memory (Lemma 10)

$$M_P(r) \leq \delta \binom{|T|}{2} \text{ qubits}$$

where δ is the diameter of the graph.

Two Dimensional Grid

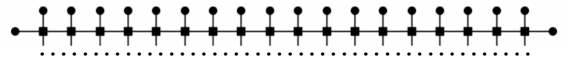


In general, the quantum memory required by a repeater r (Corollary 16)

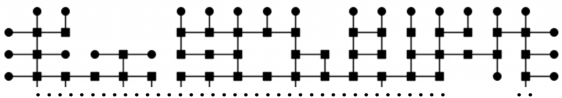
$$M(r) \in \Omega(k^2) \text{ qubits.}$$

Simulation Results

- Assumption 1: Path establishment for all terminals
  - End-to-end paths from every terminal to any other terminal:



- Assumption 2: Random arrangement of repeaters using Bernoulli bond percolation
  - Probability p of ensuring repeater connectivity greater than 0.5



- NetworkX library[1] to conduct Monte Carlo simulations[2]
- A (step-by-step) construction example follows

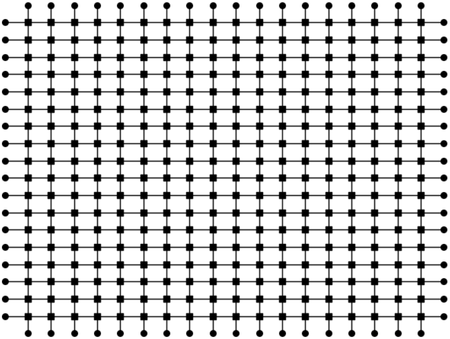
---

[1] Python Library available online at: https://networkx.github.io

[2] Code available online at: http://j.mp/QCECodeGitHub

Simulation Example 1/3

```
## Initial Parameters
k = 20 #k quadratic (2D) lattice
p = 1 #bernoulli probability for bond percolation
q = 1 #bernoulli probability for terminal arrival

DrawGrid=True
ShowLabels=False
AdditionalRing=True
BondPercolation=False
ComputePaths=False
PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
CSVFormat=False
```

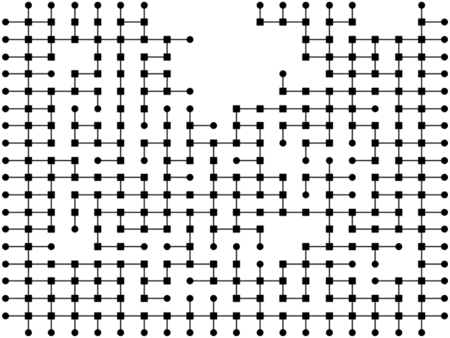


```
Output:
The graph contains 324 repeaters and 72 terminals [(k^2 (- nodes 0, 19, 380, and 399 removed, to avoid terminal adjacency]
```

Simulation Example 2/3

```
## Initial Parameters
k = 20 #k quadratic (2D) lattice
p = 0.55 #bernoulli probability for bond percolation
q = 1 #bernoulli probability for terminal arrival

DrawGrid=True
ShowLabels=False
AdditionalRing=True
BondPercolation=True
ComputePaths=False
PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
CSVFormat=False
```

Run 1

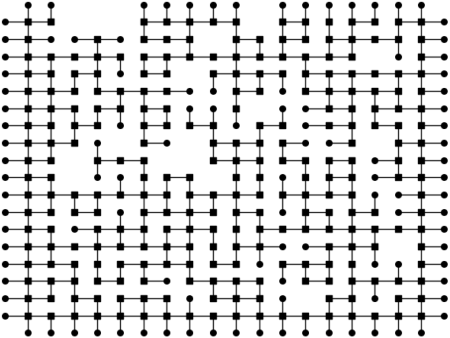


```
Output:
The graph contains 254 repeaters and 105 terminals.
```

Simulation Example 3/3

```
## Initial Parameters
k = 20 #k quadratic (2D) lattice
p = 0.55 #bernoulli probability for bond percolation
q = 1 #bernoulli probability for terminal arrival

DrawGrid=True
ShowLabels=False
AdditionalRing=True
BondPercolation=True
ComputePaths=False
PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
CSVFormat=False
```

Run 2

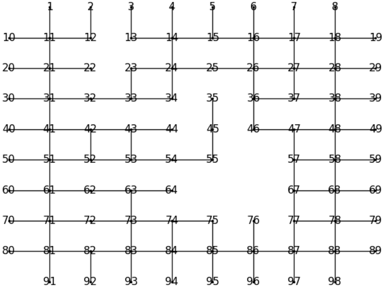


```
Output:
The graph contains 266 repeaters and 108 terminals.
```

Evaluation Example

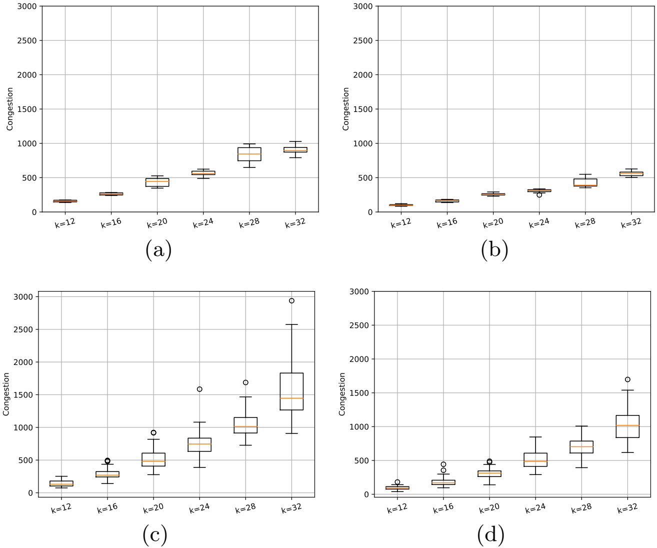
```
## Initial Parameters
k = 10 #k quadratic (2D) lattice
p = 0.65 #bernoulli probability for bond percolation
q = 1 #bernoulli probability for terminal arrival

DrawGrid=True
ShowLabels=True
AdditionalRing=True
BondPercolation=True
ComputePaths=True
PathSearchAlgorithm=1 #1=shortestPaths 2=peelingPaths
CSVFormat=False
```

```
Output:
The graph contains 56 repeaters [ [11, 12, 13, 14, 15, 16, 17,
18, 21, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 36, 37, 38, 41,
42, 43, 45, 46, 47, 48, 51, 52, 53, 54, 55, 57, 58, 61, 62, 63,
67, 68, 71, 72, 73, 74, 75, 77, 78, 81, 82, 83, 84, 85, 86, 87,
88] ] and 37 terminals [ [1, 2, 3, 4, 5, 6, 7, 8, 10, 19, 20,
29, 30, 39, 40, 49, 50, 59, 60, 69, 70, 79, 80, 89, 91, 92, 93,
94, 95, 96, 97, 98, 22, 35, 44, 64, 76] ]

Paths:
1 -> 2 : [1, 11, 12, 2]
1 -> 3 : [1, 11, 21, 31, 32, 33, 34, 24, 14, 13, 3]
...
```



```
...
22 -> 35 : [22, 21, 31, 41, 42, 43, 53, 54, 55, 45, 35]
22 -> 44 : [22, 21, 31, 41, 42, 43, 44]
22 -> 64 : [22, 21, 31, 41, 51, 61, 62, 63, 64]
22 -> 76 : [22, 21, 31, 41, 51, 61, 62, 63, 73, 74, 75, 85, 86, 76]
35 -> 44 : [35, 45, 55, 54, 53, 43, 44]
35 -> 64 : [35, 45, 55, 54, 53, 52, 51, 61, 62, 63, 64]
35 -> 76 : [35, 45, 55, 54, 53, 52, 51, 61, 62, 63, 73, 74, 75, 85, 86, 76]
44 -> 64 : [44, 43, 42, 41, 51, 61, 62, 63, 64]
44 -> 76 : [44, 43, 42, 41, 51, 61, 71, 72, 73, 74, 75, 85, 86, 76]
64 -> 76 : [64, 63, 73, 74, 75, 85, 86, 76]

Congestion =  288 (Repeater 31 appears in 288 paths, repeater 41 appears in 245 paths, repeater 51 appears in 223 paths, etc.)
Entanglement rate = 200
```

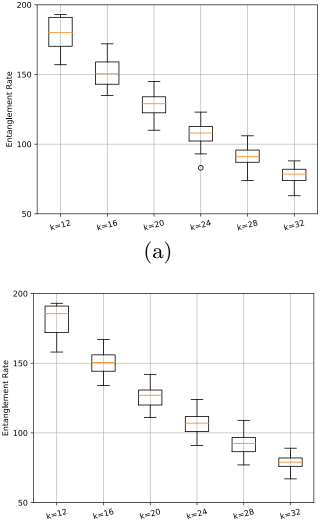
Congestion Results



(a) (b) (c) (d)

(a,c) shortest path and (b,d) peeling path strategies. Values of *p* and *q* are 0.95 in (a,b) and 0.65 in (c,d). Values of *p* and *q* are 0.95 in (a,b) and 0.65 in (c,d).

Entanglement Rate Results



(a) (b)

(c) (d)

(a,c) shortest path and (b,d) peeling path strategies. Values of p and q are 0.95 in (a,b) and 0.65 in (c,d). Values of p and q are 0.95 in (a,b) and 0.65 in (c,d).

Conclusion

- Topic: Path congestion avoidance in networks of quantum repeaters and terminals
- Assumption: Complete paths between terminals
- Evaluation
  - shortest-path establishment vs. layer-peeling path establishment
- Main results:
  - Both strategies provide an equivalent entanglement rate
  - Layer-peeling establishment considerably reduces congestion

    → Repeaters in the inner layers get less congested and would require a lower number of qubits, while providing a similar entanglement rate

Thank you! Questions?

References

- [24] M. Caleffi, Optimal routing for quantum networks, IEEE Access, 5(22):299–312, 2017.
- [25] M. Uphoff et al., Integrated quantum repeater at telecom wavelength, Applied Physics B, 122(3):46, 2016.
- [26] Y. Wang et al., Single-qubit quantum memory, Nature Photonics, 11(10):646–650, 2017.